Gauge equivalence of control systems and conjugate points

Bronisław Jakubczyk
Institute of Mathematics
Polish Academy of Sciences, Warsaw

Abstract

The class of control-affine systems

$$\dot{x} = X(x) + \sum u_i Y_i(x),$$

is preserved under invertible (gauge) transformations $u \mapsto \tilde{u} = G(x)u$ and transforming the state by a diffeomorphism $x \mapsto \tilde{x} = \Phi(x)$. Such transformations preserve the drift X and the control distribution $V = \operatorname{span}\{Y_1, \ldots, Y_m\}$, up to the diffeomorphism Φ, thus they do not change the basic features of the system. The corresponding equivalence problem reduces to the equivalence of the pairs (X, V) up to state diffeomorphisms.

Such local equivalence problem and finding its invariants is understood only for special classes of systems. We will restrict the discussion to classes of pairs (X, V) which resemble systems defined by systems of second order ODEs, un particular by Euler-Lagrange equations and semi-Hamiltonian equations. Invariants as Jacobi and Kosambi curvature or covariant differentiation can be defined and applied to analysing the behaviour of trajectories of the drift X. The notion of conjugate points used in Riemannian geometry (where X is the geodesic spray) and in optimal control has an analog here and can be analysed without assuming that the distribution V is integrable.

One feature to be discussed in the lecture will be the focussing and dispersing properties of the trajectories of the drift (the control switched off), existence of conjugate points and the possibility of periodical deflections and returns from and to the reference trajectory of X by impulsive controls. Examples of applications will be presented.

The results are joint work with W. Kryński.