Constructing flat inputs for two-output systems

F. Nicolau, W. Respondek and J.P. Barbot

Consider the following nonlinear observed dynamics:

$$\Sigma : \begin{cases} \dot{x} &= f(x), \quad x \in \mathbb{R}^n, \\ y &= h(x), \quad y \in \mathbb{R}^m. \end{cases}$$

The problem that we are studying is to find control vector fields $g_1, \ldots, g_m$ (or, equivalently, to place the actuators or the inputs) such that the control-affine system associated to Σ:

$$\Sigma_c : \dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) u_i,$$

is flat with the original measurements $(h_1, \ldots, h_m)$ being a flat output. The construction of a flat output can be seen as a problem of sensor placement in order to achieve flatness of the resulting input-output system. Dual to this, one can consider the problem of an actuator placement (i.e., of finding control vector fields $g_1, \ldots, g_m$) in order to achieve the same property. This dual problem has been recently introduced by Waldherr and Zeitz [4, 5] who call inputs $u_1, \ldots, u_m$ multiplying, respectively, $g_1, \ldots, g_m$, as *flat inputs* (which are objects dual to flat outputs).

In the single-output case, a flat input can be constructed if and only if the system Σ together with its output h is observable, see [4]. In the multi-output case, observability is not necessary for the construction of flat inputs. The observable case has been solved in [5]. The goal of this presentation is thus to treat the unobservable one and we will consider the case of two-output systems (i.e., $m = 2$), see [1, 2]. Here we deal with unobservable uncontrolled system that become at least locally weakly observable due to a suitable design of flat inputs. We show that locally there always exist control vector fields g_1 and g_2 such that the control-affine system Σ_c is flat with (h_1, h_2) being a flat output. The link between the observed (via the given outputs h_i) subsystem and the unobserved one is made with the help of flat inputs and some linking terms. We discuss how these linking terms can be chosen and address the problem of the minimal modification of the initial dynamical system Σ (the measure of modification being the number of equations that we have to change by adding inputs). Finally, we explain how our results can be applied to secure communication [3].

References

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